

Geometry I, 2006 : Coursework 2

Show your calculations

1. [unassessed] Do exercise 2 on page 3, exercise 3 on page 4, exercise 4 on page 5 and exercise 6 on page 8 of the course text, and check your answers in the back of the book.

2. [2 marks] In each of the following cases find the length $|\underline{r}|$ of $\underline{r}$, and the unit vector $\hat{r}$ in the direction of $\underline{r}$.

$$(i) \quad \underline{r} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}; \quad (ii) \quad \underline{r} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}.$$

3. (i) [3 marks] Find the vector equation, the parametric equations and the Cartesian equations for the line through the point $(2, -1, 3)$ in the direction of the vector $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$.

(ii) [1 mark] Is the point $(0, 3, 10)$ on this line? Why or why not?

4. (i) [3 marks] Find the vector equation, the parametric equations and the Cartesian equations for the line through the points $(-2, 5, 4)$ and $(2, 5, 11)$.

(ii) [1 mark] Is the point $(-6, 5, 3)$ on this line? Why or why not?

(iii) [1 mark] Is the point $(-6, 5, -3)$ on this line? Why or why not?

5. (i) [2 marks] Let $\underline{u}$ and $\underline{v}$ be any vectors. Define what is meant by the *scalar product* $\underline{u} \cdot \underline{v}$ of $\underline{u}$ and $\underline{v}$, and what it means for $\underline{u}$ and $\underline{v}$ to be *orthogonal*.

(ii) [1 mark] Determine a non-zero vector $\underline{u}$ orthogonal to $\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$.

(iii) [2 marks] Prove that if $\underline{u}$ is vector such that $\underline{u} \cdot \underline{v} = 0$ for every vector $\underline{v}$, then $\underline{u} = \underline{0}$.

6. Let $\underline{u}$ and $\underline{v}$ be non-zero vectors.

We say that $\underline{u}$ is *parallel to* $\underline{v}$ if $\underline{u} = \alpha \underline{v}$ for some (non-zero) scalar α .

(i) [2 marks] Prove that if $\underline{u}$ is parallel to $\underline{v}$ then $|\underline{u} \cdot \underline{v}| = |\underline{u}||\underline{v}|$.

(ii) [2 marks] Prove that if $\underline{u}$ is not parallel to $\underline{v}$ then $|\underline{u} \cdot \underline{v}| < |\underline{u}||\underline{v}|$.

Put your name and student number on your solutions, staple them, and put them in the **Geometry I** Collecting Box in the **Basement** of the Maths Building by 5pm on Thursday 12th October.

Leonard Soicher, 05 October 2006